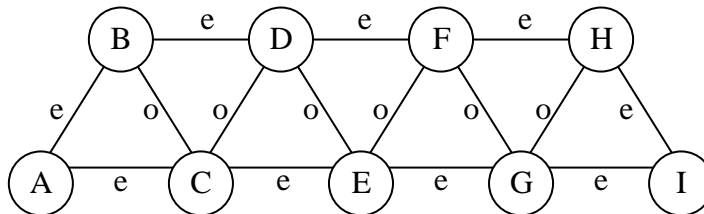


Constraint Propagation Problems

Problem 1

Consider the following constraint network, where all variables have domain $\{1,2,3\}$, except variables A and I, whose domain is $\{0,1,2,3\}$. The binary constraints labelled with e (resp. o) are satisfied if one of the variables take value 0 or the sum of the variables is even (resp. odd).

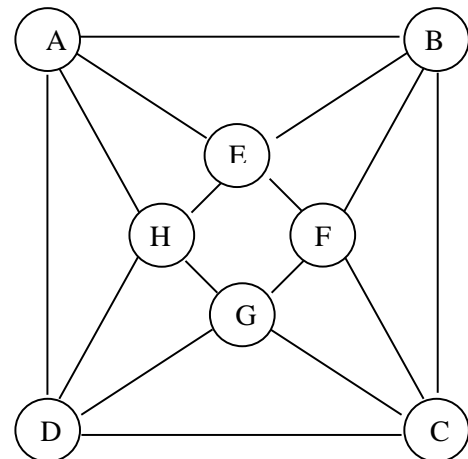


- What would be the pruning of the variables' domains if node-consistency is maintained? And arc-consistency? Justify.
- Show that path-consistency would be able to fix the value of some variables? Which ones?
- Justify whether maintaining arc-consistency would be sufficient to obtain solutions of the problem without backtracking. And path consistency?

Problem 2

Consider the following constraint network where variables A, B, C and D have domain $\{1,2,3\}$ and variables E, F, G and H have domain $\{2,3,4\}$. The binary constraints shown are all difference constraints (\neq).

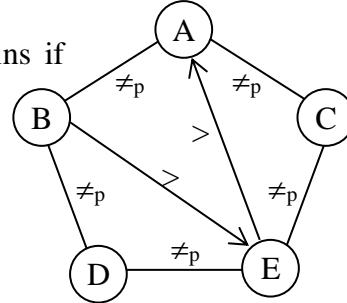
- Show that the constraint network is node- and arc-consistent. Justify.
- Assume arc-consistency is maintained on the constraint network. What would be the result of propagation once A is set to 2?
- Assume now that variables A and B are restricted to the domain $\{2,3\}$ (and the others keep their previous domains. Show that the problem becomes impossible.
- Do you think this impossibility would be obtained, without backtracking, by arc-consistency? And path consistency? Justify.



Problem 3

Consider the following constraint network, where all variables have domain $\{1,2,3,4\}$. The binary constraints labelled with \neq_p are satisfied if the constrained variables have different parity (i.e. one is even and the other odd). The binary constraints represented by directed arcs $A \rightarrow B$ should be read as $X > Y$. For the following questions provide the answers and the adequate justifications.

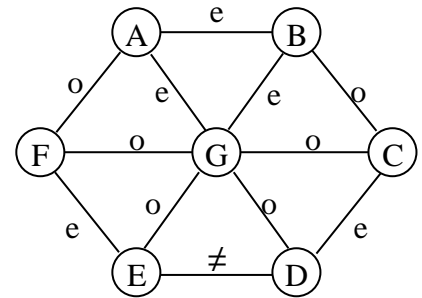
- Is the constraint network satisfiable?
- What would be the pruning of the variable domains if bounds-consistency is maintained?
- And arc-consistency?
- And path-consistency?



Problem 4

Consider the following constraint network, where nodes correspond to variables and edges to binary constraints. Edges labelled with **e** and **o** denote, respectively, constraints imposing that the sum of the variables is even and odd. Edges labelled with a \neq correspond to the usual difference constraint. All variables have domain $\{1,2,3\}$.

- What is the domain pruning achieved when node-, arc- and path-consistency is maintained? Justify.
- Consider now that the constraint between variables A and B becomes of type “o”. Verify that the problem becomes unsatisfiable and show what type of consistency should be maintained to detect such unsatisfiability without labelling the variables.

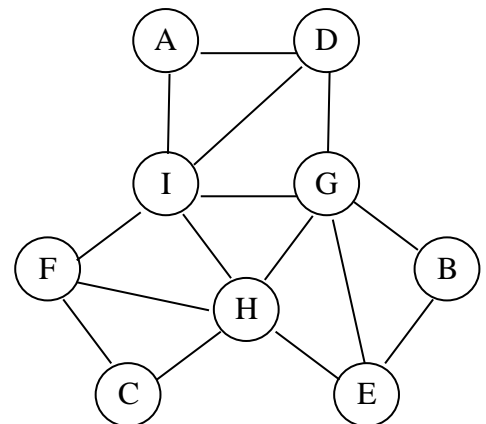


- For a generic constraint network with the above topology, where enumeration was needed, what variable ordering would you use to do such enumeration? How many variables should be labelled, to reach a backtracking free labelling of the remaining network? Justify.

Problem 5

Consider the following constraint network, where all variables have domain $\{1,2,3\}$ and the binary constraints are constraints of difference (\neq).

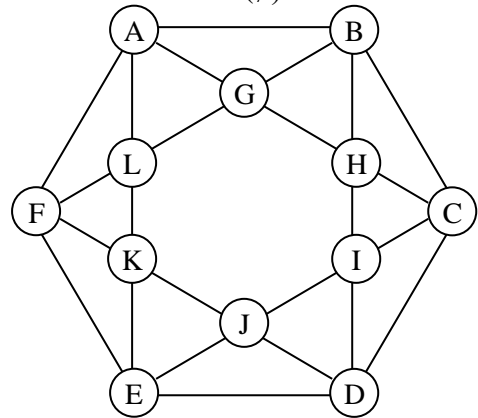
- Show that no type of consistency (node-, arc- or path-) may prune the domains of the variables. Suggestion: analyse the solutions of the problem.
- Assume that variables are labelled according to the ordering [A, B, C, D, E, F, G, H, I]. Show that maintenance of arc-consistency does not guarantee a backtracking free labelling. And path-consistency? Justify your answer.
- Present an ordering of the variables, static or dynamic, that guarantees a backtracking free search in conjunction with arc consistency. Justify your answer.
- Could you infer extra-constraints of equality ($=$) for the problem (keeping equivalence)? Should you make these constraints explicit? Why?



Problem 6

Consider the following constraint network where all the “external” variables (A to F) have domain {1,2,3} and the others domain {1,2}. All constraints are constraints of difference (\neq).

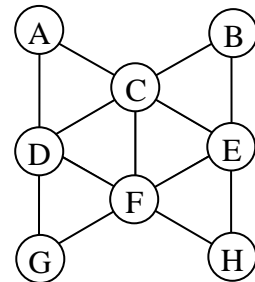
- Show that the constraint network is not satisfiable.
- Show that mere maintenance of node- and arc-consistency would not detect such unsatisfiability.
- And path-consistency?
- Show that the problem becomes satisfiable if all variables have initial domain {1,2,3}.
- Does any type of consistency achieve domain reduction for any of the variables?
- Explain what type of consistency and variable heuristics seem more adequate to this network. Will the combination avoid backtracking during search?
- How many solutions does the problem have? What additional constraints could be imposed to avoid symmetries? Justify.



Problem 7

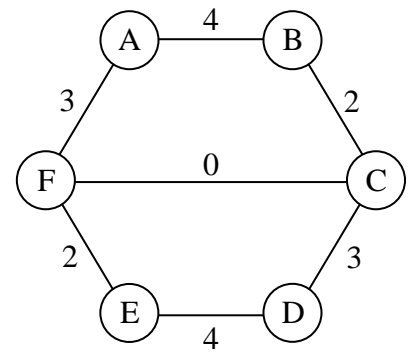
Consider the constraint network below, where nodes correspond to variables and arcs to binary constraints.

- Assuming that all constraints are constraints of difference (\neq), show that the network has no solutions if all variables have domain {1,2}. What type of consistency (node-, arc-, path- or higher k- consistency) would allow this conclusion to be inferred, without enumeration of the variables?
- Now assume that variables C and F have domain {1,3} and all other variables have domain {1,2,3}.
 - Show that maintaining arc-consistency does not prune the domain of any of the variables.
 - In contrast, show that maintaining path-consistency grounds (fixes the value of) some variables. Which variables and what are their values? Are there any other domains pruning?
 - Show that the (non-binary) constraint $A+B+G+H = 9$ is not satisfiable. Would path-consistency on the binary constraints be sufficient to infer this unsatisfiability?
- Assume now that i. all binary constraints in the network are arbitrary, ii. that all variables have d values in their domain, and iii. arc-consistency is maintained.
 - Present a static ordering of the variables that should lead to an efficient resolution of the problem. Justify your answer.
 - Assuming the same ordering for variable labelling, can you present a weaker type of consistency that, if maintained, would lead to no more backtracks during the labelling of the variables? Justify.



Problem 8

Consider the following constraint network, where all variables have domain $\{1,2,3,4,5,6\}$. The binary constraints, labelled with an integer k , are satisfied if this is the absolute distance between the constrained variables (i.e. if $k = 2$ and one variable is 3, the other must have values 1 or 5. In particular equality is obtained with $k=0$). For the following questions provide the answers and the adequate justifications.



- What would be the pruning of the variable domains if node-consistency is maintained?
- And bounds-consistency?
- And arc-consistency?