Constraint Propagation Problems

Problem 1

Consider the following constraint network, where all variables have domain {1,2,3}, except variables A and I, whose domain is {0,1,2,3}. The binary constraints labelled with e (resp. o) are satisfied if one of the variables take value 0 or the sum of the variables is even (resp. odd).

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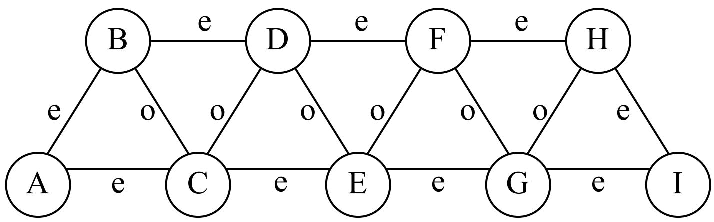


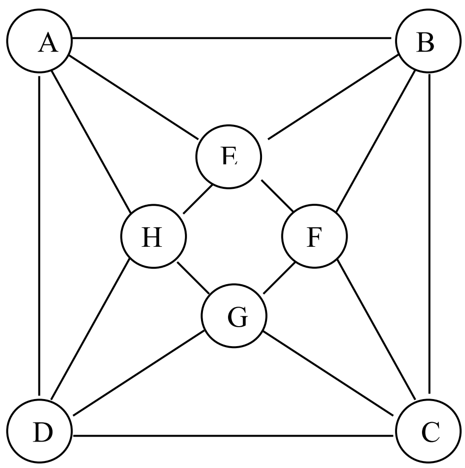
a) What would be the pruning of the variables' domains if node-consistency is maintained? And arc-consistency? Justify.

b) Show that path-consistency would be able to fix the value of some variables? Which ones?

c) Justify whether maintaining arc-consistency would be sufficient to obtain solutions of the problem without backtracking. And path consistency?

Problem 2

Consider the following constraint network where variables A, B, C and D have domain {1,2,3} and variables E, F, G and H have domain {2,3,4} The binary constraints shown are all difference constraints (≠).



a) Show that the constraint network is node- and arc-consistent. Justify.

b) Assume arc-consistency is maintained on the constraint network. What would be the result of propagation once A is set to 2?

c) Assume now that variables A and B are restricted to the domain {2,3} (and the others keep their previous domains. Show that the problem becomes impossible.

d) Do you think this impossibility would be obtained, without backtracking, by arc-consistency? And path consistency? Justify.

Problem 3

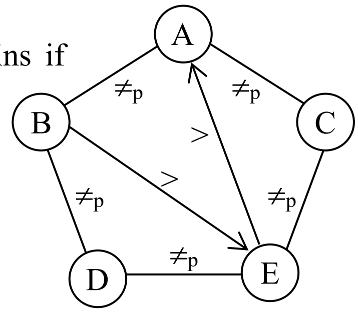
Consider the following constraint network, where all variables have domain {1,2,3,4}. The binary constraints labelled with $\neq$p are satisfied if the constrained variables have different parity (i.e. one is even and the other odd). The binary constraints represented by directed arcs $A \rightarrow B$ should be read as $X > Y$. For the following questions provide the answers and the adequate justifications.

a) Is the constraint network satisfiable?

b) What would be the pruning of the variable domains if bounds-consistency is maintained?

c) And arc-consistency?

d) And path-consistency?

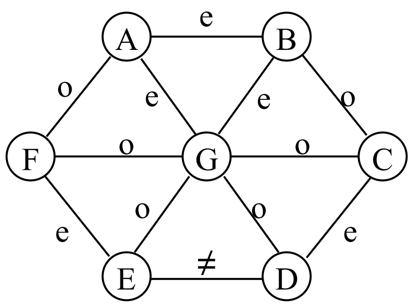


Problem 4

Consider the following constraint network, where nodes correspond to variables and edges to binary constraints. Edges labelled with **e** and **o** denote, respectively, constraints imposing that the sum of the variables is even and odd. Edges labelled with a $\neq$ correspond to the usual difference constraint. All variables have domain {1,2,3}.

a) What is the domain pruning achieved when node-, arc- and path-consistency is maintained? Justify.

b) Consider now that the constraint between variables A and B becomes of type "o". Verify that the problem becomes unsatisfiable and show what type of consistency should be maintained to detect such unsatisfiability without labelling the variables.



c) For a generic constraint network with the above topology, where enumeration was needed, what variable ordering would you use to do such enumeration? How many variables should be labelled, to reach a backtracking free labelling of the remaining network? Justify.

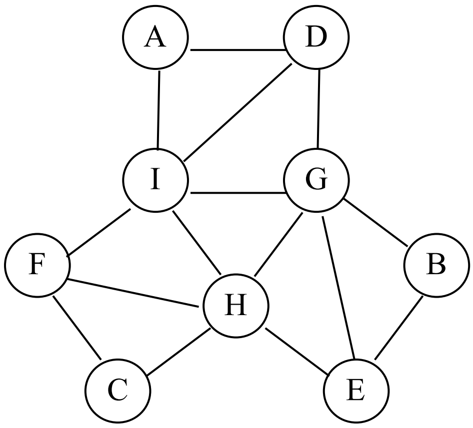
Problem 5

Consider the following constraint network, where all variables have domain {1,2,3} and the binary constraints are constraints of difference ($\neq$).

a) Show that no type of consistency (node-, arc- or path-) may prune the domains of the variables. Suggestion: analyse the solutions of the problem.

b) Assume that variables are labelled according to the ordering [A, B, C, D, E, F, G, H, I]. Show that maintenance of arc-consistency does not guarantee a backtracking free labelling. And path-consistency? Justify your answer.

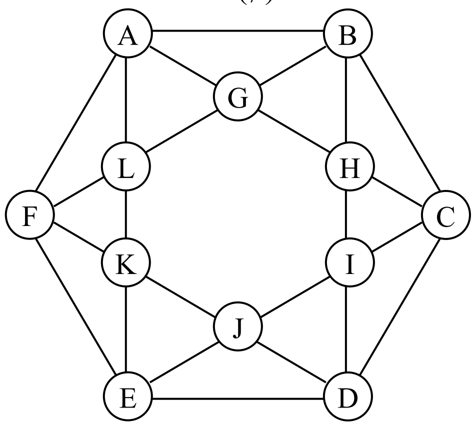
c) Present an ordering of the variables, static or dynamic, that guarantees a backtracking free search in conjunction with arc consistency. Justify your answer.



d) Could you infer extra-constraints of equality (=) for the problem (keeping equivalence)? Should you make these constraints explicit? Why?

Problem 6

Consider the following constraint network where all the "external" variables (A to F) have domain {1,2,3} and the others domain {1,2}. All constraints are constraints of difference (≠).



a) Show that the constraint network is not satisfiable.

b) Show that mere maintenance of node- and arc-consistency would not detect such unsatisfiability.

c) And path-consistency?

d) Show that the problem becomes satisfiable if all variables have initial domain {1,2,3}.

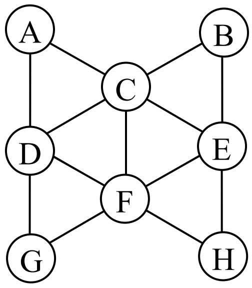
e) Does any type of consistency achieve domain reduction for any of the variables?

f) Explain what type of consistency and variable heuristics seem more adequate to this network. Will the combination avoid backtracking during search?

g) How many solutions does the problem have? What additional constraints could be imposed to avoid symmetries? Justify.

Problem 7

Consider the constraint network below, where nodes correspond to variables and arcs to binary constraints.



a) Assuming that all constraints are constraints of difference (≠), show that the network has no solutions if all variables have domain {1,2}. What type of consistency (node-, arc-, path- or higher k- consistency) would allow this conclusion to be inferred, without enumeration of the variables?

b) Now assume that variables C and F have domain {1,3} and all other variables have domain {1,2,3}.

i. Show that maintaining arc-consistency does not prune the domain of any of the variables.

ii. In contrast, show that maintaining path-consistency grounds (fixes the value of) some variables. Which variables and what are their values? Are there any other domains pruning?

iii. Show that the (non-binary) constraint A+B+G+H = 9 is not satisfiable. Would path-consistency on the binary constraints be sufficient to infer this unsatisfiability)

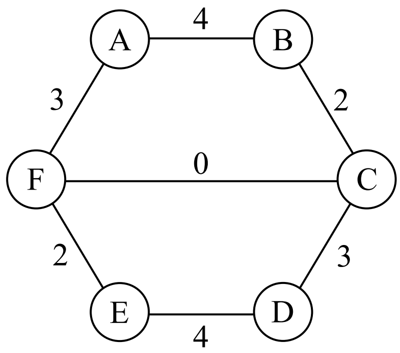
c) Assume now that i. all binary constraints in the network are arbitrary, ii. that all variables have d values in their domain, and iii. arc-consistency is maintained.

i. Present a static ordering of the variables that should lead to an efficient resolution of the problem. Justify your answer.

ii. Assuming the same ordering for variable labelling, can you present a weaker type of consistency that, if maintained, would lead to no more backtracks during the labelling of the variables? Justify.

Problem 8

Consider the following constraint network, where all variables have domain {1,2,3,4,5,6}. The binary constraints, labelled with an integer k, are satisfied if this is the absolute distance between the constrained variables (i.e. if k = 2 and one variable is 3, the other must have values 1 or 5. In particular equality is obtained with k=0). For the following questions provide the answers and the adequate justifications.



a) What would be the pruning of the variable domains if node-consistency is maintained?

b) And bounds-consistency?

c) And arc-consistency?

d) And path-consistency?