Problem Solving

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Problem Solving

Modelling Techniques

- Dependency Reduction
- Variable Elimination
- System Scaling

Some Languages and Tools

- Interval Libraries
- Constraint Solving Systems

Some Problems

- Some Common Benchmarks

Final Project

Modelling Techniques

Frequently a single problem may be modelled by several equivalent CCSPs

The behaviour of a constraint solver may change drastically even with equivalent CCSPs

To choose the *best* model for a particular problem is important to understand the underlying constraint propagation algorithms

Some modelling techniques are commonly adopted for improving the accuracy and efficiency of the continuous constraint solvers

Modelling Techniques

Dependency Reduction

A fundamental problem of interval arithmetic is the dependency problem (see lecture 2).

> **Dependency Problem.** In the interval arithmetic evaluation of an interval expression, each occurrence of the same variable is treated as a different variable. The dependency between the different occurrences of a variable in an expression is lost. ❑

Some expressions may be rewritten into equivalents that minimize the dependency problem

Examples:

- Factorize as much as possible polynomial expressions:
  - Instead of using constraint $x^2y^2+xy^2+xy=0$ use constraint $xy(y(x+1)+1)=0$
- Use better interval extensions (mean value form, Taylor form,…):
  - Instead of using constraint $x-x^2=0$ use constraint $0.25-(x-0.5)^2=0$

Modelling Techniques

Variable Elimination

Continuous constraint solvers rely on the efficiency of branch and prune algorithms for enforcing consistency on the CCSP variables

Precision and efficiency may be improved if the number of variables is reduced

Sometimes a set of constraints may be rewritten into an equivalent set with less variables

Example:

Instead of using the constraint system:

$$\begin{cases} x_1 + x_2 + x_3 = -1 \\ (x_1 + x_1 x_2 + x_2 x_3)\, x_4 = c_1 \\ (x_2 + x_1 x_3)\, x_4 = c_2 \\ x_3 x_4 = c_3 \end{cases}$$

Consider $x_4 = c_3/x_3$ and use the constraint system:

$$\begin{cases} x_1 + x_2 + x_3 = -1 \\ (x_1 + x_1 x_2 + x_2 x_3) c_3 = c_1 x_3 \\ (x_2 + x_1 x_3) c_3 = c_2 x_3 \end{cases}$$

Modelling Techniques

System Scaling

Continuous constraint solvers rely on interval techniques for dealing with numerical errors.

A consequence of numerical errors is the amplification of the variable domains and poor pruning results

Two major sources of numerical errors are:

- operations with large numbers (lower density of F-Numbers at this ranges)
- operands with different magnitudes

Scaling the system and making some variable substitutions may avoid such situations as much as possible

Example:

Instead of using constraint: $10^{-20}x^2+3x+2\times10^{20}=0$

Consider $x = 10^{20}y$ and use the constraint: $y^2+3y+2=0$

Some Languages and Tools

Interval Libraries

C

FILIB (Fast Interval LIBrary)

http://www2.math.uni-wuppertal.de/~xsc/software/filib.html

Java

IA_MATH

http://interval.sourceforge.net/interval/java/ia_math/README.html

Octave

GNU Octave interval package

https://octave.sourceforge.io/interval/

Python

pyinterval · PyPI

https://pypi.org/project/pyinterval/

Some Languages and Tools

Interval Libraries

C++

Boost Interval Arithmetic Library

https://www.boost.org/doc/libs/1_68_0/libs/numeric/interval/doc/interval.htm

PROFIL/BIAS (Programmer's Runtime Optimized Fast Interval Library)
http://www.ti3.tu-harburg.de/Software/PROFILEnglisch.html

FILIB++ (Fast Interval LIBrary in C++)

http://www2.math.uni-wuppertal.de/~xsc/software/filib.html

GAOL: NOT Just Another Interval Library

http://frederic.goualard.net/#research-software

Some Languages and Tools

Constraint Solving Systems

CLIP (Prolog)

http://interval.sourceforge.net/interval/prolog/clip/README.html

RealPaver (C++)

http://pagesperso.lina.univ-nantes.fr/~granvilliers-l/realpaver/

Elisa (C++)

http://sourceforge.net/projects/elisa

Some Problems

Some Common Benchmarks

Many continuous problems may be modelled and solved as CCSPs.

Several benchmarks are commonly used to address the quality of the constraint solvers.

The following problems illustrate some common benchmarks in interval constraints.

For each benchmark we would like to know where are the solutions (if any) satisfying the constraints within the specified ranges

Some Problems

Bronstein

Constraints:

$$x^2 + y^2 + z^2 = 36$$
$$x + y = z$$
$$xy + z^2 = 1$$

Ranges:

$$x, y, z \in [-10^8, 10^8]$$

Some Problems

Chem

Constraints:

$R = 10$
$R_5 = 0.193$
$R_6 = 0.002597/\sqrt{40}$
$R_7 = 0.003448/\sqrt{40}$
$R_8 = 0.00001799/40$
$R_9 = 0.0002155/\sqrt{40}$
$R_{10} = 0.00003846/40$
$3x_5=x_1(x_2+1)$
$x_2(2x_1+x_3^2+R_8+2R_{10}x_2+R_7x_3+R_9x_4)+x_1=Rx_5$
$x_3(2x_2x_3+2R_5x_3+R_6+R_7x_2)=8x_5$
$x_4(R_9x_2+2x_4)=4Rx_5$
$x_2(x_1+R_{10}x_2+x_3^2+R_8+R_7x_3+R_9x_4)+x_1+x_3(R_5x_3+R_6)+x_4^2=1$

Ranges:

$x_1, x_2, x_3, x_4, x_5 \in [0,10^8]$

Some Problems

Chemk

Constraints:

$$x_1^2 = x_2$$
$$x_4^2 = x_3$$
$$2.177e^7 x_2 + 0.55 x_1 x_4 + 0.45 x_1 = x_4 + 1.697e^7 x_2 x4$$
$$1.585\times10^{14} x_2 x_4 + 4.126\times10^7 x_1 x_3 + 2.284\times10^7 x_3 x_4 + 48.4 x_4 =$$
$$8.282\times10^6 x_1 x_4 + 1.918\times10^7 x_3 + 27.73$$

Ranges:

$$x_1, x_2, x_3, x_4 \in [0,1]$$

Some Problems

Cyclo

Constraints:

$$y^2z^2 + y^2 + z^2 + 13 = 24yz$$
$$x^2z^2 + x^2 + z^2 + 13 = 24xz$$
$$x^2y^2 + x^2 + y^2 + 13 = 24xy$$

Ranges:

$$x, y, z \in [0, 10^5]$$

Some Problems

Combustion

Constraints:

$$x_2 + 2x_6 + x_9 + 2x_{10} = 10^{-5}$$
$$x_3 + x_8 = 3\times10^{-5}$$
$$x_1 + x_3 + 2x_5 + 2x_8 + x_9 + x_{10} = 5\times10^{-5}$$
$$x_4 + 2x_7 = 10^{-5}$$
$$0.5140437\times10^{-7}x_5 = x_1^2$$
$$0.1006932\times10^{-6}x_6 = 2x_2^2$$
$$0.7816278\times10^{-15}x_7 = x_4^2$$
$$0.1496236\times10^{-6}x_8 = x_1x_3$$
$$0.6194411\times10^{-7}x_9 = x_1x_2$$
$$0.2089296\times10^{-14}x_{10} = x_1x_2^2$$

Ranges:

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} \in [-10^8, 10^8]$$

Some Problems

Freudenstein

Constraints:

$$x_1 + 5x_2^2 = x_2^3 + 2x_2 + 13$$
$$x_1 + x_2^3 + x_2^2 = 14x_2 + 29$$

Ranges:

$$x_1, x_2 \in [-10^8, 10^8]$$

Some Problems

Himmelblau

Constraints:

$$2x_2^2 + 4x_1x_2 + 4x_1^3 = 42x_1 + 14$$
$$2x_1^2 + 4x_1x_2 + 4x_2^3 = 26x_2 + 22$$

Ranges:

$$x_1, x_2 \in [-10^8, 10^8]$$

Some Problems

I1

Constraints:

$$x_1 = 0.25428722 + 0.183247757\, x_4\, x_3\, x_9$$
$$x_2 = 0.37842197 + 0.16275449\, x_1\, x_{10}\, x_6$$
$$x_3 = 0.27162577 + 0.16955071\, x_1\, x_2\, x_{10}$$
$$x_4 = 0.19807914 + 0.15585316\, x_7\, x_1\, x_6$$
$$x_5 = 0.44166728 + 0.19950920\, x_7\, x_6\, x_3$$
$$x_6 = 0.14654113 + 0.18922793\, x_8\, x_5\, x_{10}$$
$$x_7 = 0.42937161 + 0.21180486\, x_2\, x_5\, x_8$$
$$x_8 = 0.07056438 + 0.17081208\, x_1\, x_7\, x_6$$
$$x_9 = 0.34504906 + 0.19612740\, x_{10}\, x_6\, x_8$$
$$x_{10} = 0.42651102 + 0.21466544\, x_4\, x_8\, x_1$$

Ranges:

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} \in [-2,2]$$

Some Problems

Kincox

Constraints:

$$6(x_1 x_2 - x_3 x_4) + 10 x_1 = 1$$
$$6(x_1 x_4 + x_2 x_3) + 10 x_3 = 4$$
$$x_1^2 + x_3^2 = 1$$
$$x_2^2 + x_4^2 = 1$$

Ranges:

$$x_1, x_2, x_3, x_4 \in [-1,1]$$

Some Problems

Nauheim

Constraints:

$$eg + 2dh = 0$$
$$9e + 4b = 0$$
$$4ch + 2ef + 3dg = 0$$
$$7c + 8f = 9a$$
$$4df + 5cg + 6h + 3e = 0$$
$$5d + 6cf + 7g - 9b = 0$$
$$9d + 6a = 5b$$
$$7a = 9c + 8$$

Ranges:

$$a, b, c, d, e, f, g, h \in [-10^8, 10^8]$$

Project

The goal of the project is to implement a simple constraint solver for processing polynomial equality constraints.

From a set of polynomial equality constraints and an initial domains box the constraint solver should identify where the solutions are.

A branch-and-prune algorithm must be implemented to maintain a set of boxes consistent with the constraints.

The pruning results from constraint propagation over a set of narrowing functions associated with the constraints.

Each narrowing function narrows the domain of a single variable based on the interval Newton method.

The constraint solver should be used to solve each of the previous benchmark problems.