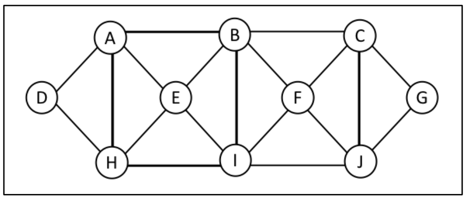
Constraint Programming

2018/2019– Mini-Test #1
Friday, 26 October, 18:00 h, Room 204-II
Duration: 1.5 h (open book)

1. Finite domain Constraints - Propagation (7 pts)



Consider the constraint network on the right, where nodes represent variables, all with domain {**1,2,3**}. Arcs represent constraints of difference (i.e. $X \neq Y$).

a) **(2 pt)** Is the problem satisfiable? If so, how many different solutions exist? Justify your answer.

b) **(1 pt)** What pruning is achieved initially, if node-consistency is maintained? And arc-consistency?

c) **(2 pt)** Are there any implicit binary constraint in the network? What type of consistency would be needed to infer such constraints? Justify your answer.

d) **(2 pt)** Since any solution to the problem has a symmetric solution that is a permutation of the values of variables A, D and H, could you impose symmetry breaking constraints to avoid obtaining symmetric solutions? With these constraints would there be any ordering of the labelling of variables that would be backtrack free? What type of consistency would there be needed? Justify your answer.

2. Modelling with Finite Domain Constraints (8 pts)

The Secret Code

Damien's grandfather left him a safe with an opening secret code of 4 digits and a card with the following codes written in it.

3702 4329 4961 5781 5781 5905 7685 8314 8764

The card also mentioned that each of these 9 codes had at least one correct digit (in the correct place), i.e. the same digit in the same place of the secret code. Can you find the code?

a) **(2 pt)** Specify a model for this problem in Comet. First, declare the data structures being used, including the decision variables you adopt as well as their domains. Assume a generalization of the problem, where the secret code has n digits (n=4 in this case) and there are m different codes (m=8 in this case), and these are provided in a m*n matrix (an 8*4 matrix, here) of digits.

b) **(4 pt)** To complete the models, what constraints would you consider in your model? Would you use any specific heuristics to label the variables? Justify your answer.

c) **(2 pt)** Assuming that there are more than one secret codes that satisfy the constraints, how would you change your model to obtain the largest such code (assuming the codes are read as integer numbers, i.e. code 7685 > 7641).

3. Global Constraints (5 pts)

Consider a "global" constraint that enforces two arrays (assumed to be indexed by the same range, thus having the same number of elements) to have no repeated values, and sharing exactly **k** values in their elements, not necessarily in the same positions. For example, the arrays `x = [2,5,7,9]` and `y = [9,5,7,2]` satisfy this constraint for `k = 2,` since values `2` and `7` are assigned to variables of both arrays and no other elements of **x** (resp. `y`) appear in **y** (resp. **x**). In contrast, **x** and `z = [3,7,1,7]` do not satisfy the constraint, not only because there is only one shared value (`7`) but also because it is repeated in **z**.

a) **(2pt)** Implement in Comet this constraint by means of a function declared as

```
function void share(var<CP>{int} k, var<CP>{int} [] x, var<CP>{int} []y){…}
```

b) **(2 pt)** If your implementation of this global constraint maintained domain consistency, what values would be pruned from the domain of the variables **x** and **y** if their domains are

	`x[1] in 1..3`	`y[1] in 1..8`
	`x[2] in 1..3`	`y[2] in 2..4`
`k in 0..3`	`x[3] in 1..3`	`y[3] in 2..4`
	`x[4] in 2..6`	`y[4] in 2..4`
	`x[5] in 3..6`	`y[5] in 3..7`

c) **(1 pt)** In the same conditions of the previous item, assume that constraint `y[1] < 5` is posted. Would there be any further pruning of the domains?