Constraint Programming / Programação com Restrições

2015/2016 – 2nd Test - Interval Constraints

Friday, 18 December, 14:00 h

Duration: 2 h (open book)

1. Interval Arithmetic

1.1. Prove the inclusion monotonicity property of the interval arithmetic addition operator.

1.2. Define an algorithm for the interval multiplication minimizing the number of multiplication of bounds using a case-based definition depending on the signs of intervals.

1.3. Define an algorithm for the interval power operation given positive integer exponents.

1.4. Prove that the natural interval evaluation of

$$f(x) = (x-1)(x-4)$$

is exact for intervals greater than 4.

2. Interval Newton

2.1. Use the interval Newton method to prove that there exists a unique root of

$$f(x) = x^2 - 2$$

in [1,2].

2.2. Let $x \in [-1, 1]$ and

$$f(x) = \frac{x^3}{3} - \frac{x^2}{2} + \frac{x}{2} + 1$$

a. Compute the natural interval extension of $f'(x)$ and show the results obtained with one iteration of the interval Newton method with this extension.

b. Compute the results obtained with one iteration of the interval Newton method using the expression $f'(x) = (x-0.5)^2 + 0.25$

c. Compare and discuss both approaches.

3. Constraint Propagation

3.1. Consider the constraint $x = \sqrt{x}$ with $x \in [0.5, 2]$

a. Find narrowing functions for the constraint by decomposing it into primitive constraints.

b. The successive application of the above narrowing functions up to a fixed-point converges to an interval *X*. Define *X* and justify your answer.

c. Characterize the interval of convergence computed by the direct application of the interval Newton method. Justify.